

Masses of spin-1 mesons and dynamical chiral symmetry breaking

Bing An Li

Department of Physics and Astronomy, University of Kentucky

Lexington, KY 40506, USA

Abstract

In terms of an effective chiral theory of mesons it is shown quantitatively that in the limit of $m_q = 0$, the masses of ρ , ω , $K^*(892)$, m_ϕ , a_1 , $f_1(1286)$, $K_1(1400)$, and $f_1(1510)$ mesons originate from dynamical chiral symmetry breaking.

The origin of the masses of hadrons is always one of the most important topics in hadron dynamics. The mass of a hadron is associated with chiral symmetry breaking. There are implicit chiral symmetry breaking from the current quark masses and dynamical chiral symmetry breaking. The Nambu-Jona-Lasinio model[1], starting from massless quark, has non-vanishing quark condensate which means dynamical chiral symmetry breaking. On the other hand, dynamical chiral symmetry breaking in Quantum Chromodynamics(*QCD*) has been studied extensively[2]. *QCD* has dynamical chiral symmetry breaking. It is well known that in the chiral limit, the octet pseudoscalar mesons are Goldstone mesons and massless. From the theory of chiral symmetry breaking Gell-Mann, Oakes, and Renner[3] have obtained

$$m_\pi^2 = -\frac{2}{f_\pi^2}(m_u + m_d) < 0|\bar{\psi}\psi|0 > . \quad (1)$$

The smallness of the masses of the current quarks leads to the light pseudoscalar mesons. The heavy η' meson is associated with $U(1)$ problem[4]. Why the mass of the ρ meson is much heavier than pion's mass? In this letter we try to find the origin of the ρ meson's mass. In Ref.[5] we have proposed an effective chiral theory of mesons(pions, η , ρ , ω , $a_1(1260)$, and $f_1(1286)$). In the case of two flavors the Lagrangian of this theory is constructed by using $U(2)_L \times U(2)_R$ chiral symmetry and the minimum coupling principle

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a\gamma_5 - mu(x))\psi(x) \\ & + \frac{1}{2}m_0^2(\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu) - \bar{\psi}(x)M\psi(x), \end{aligned} \quad (2)$$

where $a_\mu = \tau_i a_\mu^i + f_\mu$, $v_\mu = \tau_i \rho_\mu^i + \omega_\mu$, and $u = \exp\{i\gamma_5(\tau_i \pi_i + \eta)\}$, m is a parameter and M is the quark mass matrix. In Eq.(2) u can be written as

$$u = \frac{1}{2}(1 + \gamma_5)U + \frac{1}{2}(1 - \gamma_5)U^\dagger, \quad (3)$$

where $U = \exp\{i(\tau_i \pi_i + \eta)\}$. The introduction of the couplings between the pseudoscalar mesons and the quarks is based on the formalism of the nonlinear σ model. The Vector Meson Dominance(VMD) is a natural result of this theory, Weinberg's first sum rule is derived from this theory analytically. A unified study of the processes of normal parity and abnormal parity is presented by this theory. Most theoretical results agree with the data within about 10%. Besides the success in the phenomenology of mesons, this theory has dynamical chiral symmetry breaking(Eq.(124) of Ref.[5])

$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = 3m^3 g^2 \left(1 + \frac{1}{2\pi^2 g^2}\right), \quad (4)$$

where g is the universal coupling constant and defined as

$$g^2 = \frac{f_\pi^2}{m_\rho^2} + \frac{f_\pi^2}{6m^2}, \quad (5)$$

where f_π is the pion decay constant and $f_\pi = 186 \text{ MeV}$, $m_\rho^2 = m_0^2/g^2$. Eq.(4) means that the parameter m of the Lagrangian(Eq.(2)) is the indication of the dynamical chiral symmetry breaking. Use of the Eq.(4) leads to the mass formula of pion(Eq.(1))[5]. In the chiral limit, there are three parameters: cutoff Λ , m , and m_ρ and g is determined by the ratio Λ/m (see

Eq.(129) of Ref.[5]). The purpose of this letter is to illustrate that in the limit of $m_q = 0$, the masses of spin-one mesons are resulted by dynamical chiral symmetry breaking and m_ρ can be determined theoretically and is no longer an input. The introduction of the mass terms of the spin-1 mesons to the Lagrangian(Eq.(2)) is necessary. Otherwise, this theory cannot be well defined. For example, without these mass terms the mixing between the axial-vector meson and the corresponding pseudoscalar meson cannot be erased, the physical axial-vector and pseudoscalar fields cannot be defined. On the other hand, in the effective Lagrangian of meson fields derived from Eq.(2), except for the kinetic terms of the spin-1 mesons, the spin-1 fields only appear in the covariant derivatives(see Eq.(13) of Ref.[5])

$$D_\mu U = \partial_\mu U - i[v_\mu, U] + i\{a_\mu, U\}.$$

Because the a_μ fields are in the anticommutator, a spontaneous symmetry breaking mechanism leads to the mass difference between the axial-vector and vector mesons. However, the vector fields are in the commutator, there is no way to generate a mass term for the vector meson in this theory. Therefore, the mass terms of spin-1 mesons must be introduced.

In *QCD*, in principle, the mass of the ρ meson should be determined by a dynamical equation of bound state. In this theory KSFR sum rule[6]

$$g_\rho = \frac{1}{2}f_{\rho\pi\pi}f_\pi^2 \tag{6}$$

can be taken as the equation used to determine m_ρ . This sum rule is obtained by using

PCAC and current algebra in the limit of $p_\pi \rightarrow 0$. g_ρ is the coupling constant of $\rho - \gamma$ and $f_{\rho\pi\pi}$ is the coupling constant of $\rho \rightarrow \pi\pi$, which is defined in the limit of $p_\pi \rightarrow 0$. In Ref.[5] KSFR sum rule is satisfied numerically. On the other hand, this sum rule can be derived analytically from this theory. From Eq.(2) the currents are found to be

$$V_\mu^i = \bar{\psi} \frac{\tau_i}{2} \gamma_\mu \psi, \quad A_\mu^i = \bar{\psi} \frac{\tau_i}{2} \gamma_\mu \gamma_5 \psi.$$

These currents observe current algebra. Without the quark mass term the Lagrangian(Eq.(2)) is global $U(2)_L \times U(2)_R$ chiral symmetric and both the isovector axial-vector current are conserved. The mass term of the pion is generated from the quark mass term[7]

$$- \bar{\psi}(x) M \psi(x) = i \text{Tr} M s_F(x, x) = \frac{i}{(2\pi)^D} \text{Tr} M \int d^D p s_F(x, p), \quad (7)$$

where the quark propagator satisfies the equation[5]

$$\{\gamma \cdot (i\partial + p + v + a\gamma_5) - mu - M\} s_F(x, p) = 1. \quad (8)$$

Using the solution of Eq.(8)[5] and to the leading order in quark mass expansion, we obtain

$$- M \bar{\psi} M \psi = -\frac{1}{2} m_\pi^2 \pi_i \pi_i + \dots, \quad (9)$$

where m_π^2 is expressed by Eq.(1). With the pion mass term PCAC is derived from the Lagrangian(Eq.(2))

$$\partial^\mu A_\mu^i = \frac{1}{2} m_\pi^2 f_\pi \pi^i.$$

Therefore, the KSFR sum rule can be derived from this theory in the same way as in Ref.[6]. It is necessary to point out that due to the limit of $p_\pi \rightarrow 0$, the coupling constant $f_{\rho\pi\pi}$ of Eq.(6) is defined in nonphysical region. However, KSFR sum rule agrees with data within 10%. This fact means that in the limit of $p_\pi \rightarrow 0$, $f_{\rho\pi\pi}$ is very close to the physical one. The effective chiral theory[5] provides an explanation to this property of $f_{\rho\pi\pi}$. The expression of $f_{\rho\pi\pi}$ determined in Ref.[5] (see Eq.(48) of [5]) shows a weak dependence of $f_{\rho\pi\pi}$ on the pion momentum (in the reasonable range of the coupling constant g , the contribution of the terms which are proportional to the pion momentum is less than about 10%). In Ref.[5] $g = 0.35$ is chosen and

$$f_{\rho\pi\pi} = \frac{2}{g}, \quad (10)$$

which is independent of p_π . g_ρ is determined to be[5]

$$g_\rho = \frac{1}{2}gm_\rho^2. \quad (11)$$

Substituting Eqs.(10),(11) into the KSFR sum rule it is found

$$m_\rho^2 = 2\frac{f_\pi^2}{g^2}. \quad (12)$$

Using Eq.(5), we obtain

$$m_\rho^2 = 6m^2. \quad (13)$$

Therefore, the mass of ρ meson is no longer an input and is determined by the parameter m of Eq.(2) completely. In the limit of $m_q = 0$, there are only two parameters in the

Lagrangian(Eq.(2)), which are cutoff Λ and m . Using Eq.(4), we obtain

$$m_\rho^2 = 6 < 0|\bar{\psi}\psi|0 >^{\frac{2}{3}} (3g^2 + \frac{3}{2\pi^2})^{-\frac{2}{3}}. \quad (14)$$

In the limit of $m_q = 0$, the mass of ρ meson originates from dynamical chiral symmetry breaking. Combining Eqs.(5),(13), we obtain

$$m^2 = \frac{f_\pi^2}{3g^2} = 0.094 GeV^2, \quad < 0|\bar{\psi}\psi|0 > = -(0.247)^3 GeV^3. \quad (15)$$

The mass of ρ meson is determined to be

$$m_\rho = 0.751 GeV. \quad (16)$$

It is only 2% away from the experimental value of 0.77GeV. The numerical value of m is slightly different from the one presented in Ref.[5]. The reason is that in Ref.[5] the physical value of m_ρ is taken as input. In the limit of $m_q = 0$ ($q = u, d, s$), we should have

$$m_\phi = m_{K^*(892)} = m_\omega = m_\rho. \quad (17)$$

Therefore, in the limit of $m_q = 0$, the masses of the four low lying vector mesons originate from dynamical chiral symmetry breaking. From Eqs.(4),(12),(13) it is found that

$$f_\pi^2 = 3g^2 m^2 = 3g^2 < 0|\bar{\psi}\psi|0 >^{\frac{2}{3}} (3g^2 + \frac{3}{2\pi^2})^{-\frac{2}{3}}. \quad (18)$$

The pion decay constant is the result of dynamical chiral symmetry breaking too. Therefore, in the limit of $m_q \rightarrow 0$, the decay constants of the octet pseudoscalar mesons originate from

dynamical chiral symmetry breaking. Using Eqs.(1),(14),and (18) we obtain

$$\frac{m_\pi^2}{m_\rho^2} = -\frac{(g^2 + \frac{1}{2\pi^2})^{\frac{4}{3}}}{3^{\frac{2}{3}}g^2} \frac{m_u + m_d}{<0|\bar{\psi}\psi|0>^{\frac{1}{3}}}. \quad (19)$$

In Ref.[5] the mass relations have been found

$$(1 - \frac{1}{2\pi^2g^2})m_a^2 = 6m^2 + m_\rho^2, \quad (1 - \frac{1}{2\pi^2g^2})m_f^2 = 6m^2 + m_\omega^2, \quad (20)$$

where m_a is the mass of the meson $a_1(1260)$ and m_f is the mass of the meson $f_1(1286)$. In this theory $m_\rho = m_\omega$, the theory predicts $m_f = m_a$. Using Eq.(13), the mass relations(Eq.(20)) are rewritten as

$$m_f^2 = m_a^2 = 2m_\rho^2 \frac{g_a^2}{g_\rho^2}, \quad (21)$$

where $g_a = \frac{1}{2}m_\rho^2g^2(1 - \frac{1}{2\pi^2g^2})^{-\frac{1}{2}}$ determined in Ref.[5]. This relation is different from Weinberg's second sum rule[8], as pointed in Ref.[9], it is the result of Weinberg's first sum rule[8] and KSFR sum rule. On the other hand, the relation between m_a^2 and the quark condensate is established by using Eq.(14)

$$m_f^2 = m_a^2 = 12\frac{g_a^2}{g_\rho^2} <0|\bar{\psi}\psi|0>^{\frac{2}{3}} (3g^2 + \frac{3}{2\pi^2})^{-\frac{2}{3}}. \quad (22)$$

In the limit of $m_q = 0$, the masses of $a_1(1260)$ and $f_1(1286)$ are resulted in dynamical chiral symmetry breaking. Using $g = 0.35$, we obtain $m_f = m_a = 1388MeV$ and the experimental data are $m_a = 1230 \pm 40MeV$ and $m_f = 1282 \pm 2MeV$. The deviations are about 10%.

The mass relations have been obtained in Ref.[10] in which the theory is generalized to include the strange quark

$$(1 - \frac{1}{2\pi^2 g^2})m_{K_1(1400)}^2 = 6m^2 + m_{K^*(892)}^2, \quad (1 - \frac{1}{2\pi^2 g^2})m_{f_1(1510)}^2 = 6m^2 + m_\phi^2. \quad (23)$$

Use of Eq.(13) leads to

$$m_{K_1(1400)}^2 = \frac{g_a^2}{g_\rho^2}(m_\rho^2 + m_{K^*(892)}^2), \quad m_{f_1(1510)}^2 = \frac{g_a^2}{g_\rho^2}(m_\rho^2 + m_\phi^2). \quad (24)$$

Substituting Eq.(21) into Eq.(24) we obtain

$$m_{K_1(1400)}^2 = \frac{m_a^2}{2}(1 + \frac{m_{K^*(892)}^2}{m_\rho^2}), \quad m_{f_1(1510)}^2 = \frac{m_a^2}{2}(1 + \frac{m_\phi^2}{m_\rho^2}). \quad (25)$$

The deviations of these mass relations from the experimental values are about 3%. By using Eq.(14), the dependences of $m_{K_1(1400)}^2$ and $m_{f_1(1510)}^2$ on the quark condensate are obtained

$$\begin{aligned} m_{K_1(1400)}^2 &= 12 \frac{g_a^2}{g_\rho^2} < 0 | \bar{\psi}\psi | 0 >^{\frac{2}{3}} (3g^2 + \frac{3}{2\pi^2})^{-\frac{2}{3}} + \frac{g_a^2}{g_\rho^2}(m_{K^*(892)}^2 - m_\rho^2) \\ m_{f_1(1510)}^2 &= 12 \frac{g_a^2}{g_\rho^2} < 0 | \bar{\psi}\psi | 0 >^{\frac{2}{3}} (3g^2 + \frac{3}{2\pi^2})^{-\frac{2}{3}} + \frac{g_a^2}{g_\rho^2}(m_\phi^2 - m_\rho^2). \end{aligned} \quad (26)$$

It is well known that the mass differences of $m_{K^*(892)}^2 - m_\rho^2$ and $m_\phi^2 - m_\rho^2$ are proportional to the masses of quarks. From Eqs.(22),(26) we conclude that in the limit of $m_q = 0$, the masses of the four axial-vector mesons originate from dynamical chiral symmetry breaking.

To conclude, in the limit of $m_q \rightarrow 0$, in the effective chiral theory the masses of the eight spin-one mesons and the decay constants of the octet pseudoscalar mesons originate from dynamical chiral symmetry breaking. Three new mass relations(Eqs.(21), (24)) are found.

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References

- [1] .Nambu and G.Jona-Lasinio, Phys. Rev. **122**,345(1961); **124**,246(1961). R.Finger and J.E.Mondula, Nucl. Phys. **B199**, 168(1982).
- [2] K.Lane, Phys. Rev. **D100**, 2605(1974); H.D.Politzer, Nucl. Phys. **B117**, 397(1976); H.Pagels, Phys. Rev. **D19**, 3080(1979); V.Elias and M.D.Scadron, Phys. Rev. **D30**, 647(1984); L.N.Chang and N.P.Chang, Phys. Rev. **D29**, 312 (1984); G.Krein, P.Tang, and A.G.Williams, Phys. Lett. **B215**, 145(1988); J.Kogut et al., Phys. Rev. Lett. **48**, 1140(1982).
- [3] M.Gell-mann, R.J.Oakes, and B.Renner, Phys.Rev., **175**, 195(1968).
- [4] E. Witten, Nucl. Phys., **B149**, 285(1979); G. Veneziano, Nucl. Phys., **B159**,213(1979); C. Rosenzweig, J. Schechter, and C. G. Trahern, Phys. Rev., **D21**,3388(1980); P. Nath and R. Arnowitz, Phys. Rev., **D23**,473(1981).
- [5] Bing An Li, $U(2)_L \times U(2)_R$ Chiral Theory of Mesons, to appear in **52D** No.9(1995).
- [6] K.Kawarabayashi and M.Suzuki, Phys.Rev.Lett., **16** 255,(1966); Riazudin and Fayyazudin, Phys.Rev., **147**,1071(1966).

- [7] Bing An LI, Phys.Rev., **D50**,2243(1994).
- [8] S.Weinberg, Phys.Rev.Lett., **17**, 616(1966).
- [9] Bing An Li, Talk presented at the International Europhysics Conference on High Energy Physics(HEP95), July 27-Aug.2, Brussels, Belgium.
- [10] Bing An Li, $U(3)_L \times U(3)_R$ Chiral Theory of Mesons, to appear in **52D**, No.9(1995).